

Common statistical tests are linear models

Last updated: 28 June, 2019. Also check out the [Python version!](#)

See worked examples and more details at the accompanying notebook: <https://lindeloev.github.io/tests-as-linear>

	Common name	Built-in function in R	Equivalent linear model in R	Exact?	The linear model in words	Icon
Simple regression: $lm(y \sim 1 + x)$	y is independent of x P: One-sample t-test N: Wilcoxon signed-rank	t.test(y) wilcox.test(y)	$lm(y \sim 1)$ $lm(\text{signed_rank}(y) \sim 1)$	✓ for N > 14	One number (intercept, i.e., the mean) predicts y. - (Same, but it predicts the <i>signed rank</i> of y.)	
	P: Paired-sample t-test N: Wilcoxon matched pairs	t.test(y1, y2, paired=TRUE) wilcox.test(y1, y2, paired=TRUE)	$lm(y_2 - y_1 \sim 1)$ $lm(\text{signed_rank}(y_2 - y_1) \sim 1)$	✓ for N > 14	One intercept predicts the pairwise $y_2 - y_1$ differences. - (Same, but it predicts the <i>signed rank</i> of $y_2 - y_1$.)	
	y ~ continuous x P: Pearson correlation N: Spearman correlation	cor.test(x, y, method='Pearson') cor.test(x, y, method='Spearman')	$lm(y \sim 1 + x)$ $lm(\text{rank}(y) \sim 1 + \text{rank}(x))$	✓ for N > 10	One intercept plus x multiplied by a number (slope) predicts y. - (Same, but with <i>ranked x</i> and y)	
	y ~ discrete x P: Two-sample t-test P: Welch's t-test N: Mann-Whitney U	t.test(y1, y2, var.equal=TRUE) t.test(y1, y2, var.equal=FALSE) wilcox.test(y1, y2)	$lm(y \sim 1 + G_2)^A$ $gls(y \sim 1 + G_2, \text{weights}=\dots^B)^A$ $lm(\text{signed_rank}(y) \sim 1 + G_2)^A$	✓ ✓ for N > 11	An intercept for group 1 (plus a difference if group 2) predicts y. - (Same, but with one variance <i>per group</i> instead of one common.) - (Same, but it predicts the <i>signed rank</i> of y.)	
Multiple regression: $lm(y \sim 1 + x_1 + x_2 + \dots)$	P: One-way ANOVA N: Kruskal-Wallis	aov(y ~ group) kruskal.test(y ~ group)	$lm(y \sim 1 + G_2 + G_3 + \dots + G_N)^A$ $lm(\text{rank}(y) \sim 1 + G_2 + G_3 + \dots + G_N)^A$	✓ for N > 11	An intercept for group 1 (plus a difference if group $\neq 1$) predicts y. - (Same, but it predicts the <i>rank</i> of y.)	
	P: One-way ANCOVA	aov(y ~ group + x)	$lm(y \sim 1 + G_2 + G_3 + \dots + G_N + x)^A$	✓	- (Same, but plus a slope on x.) <i>Note: this is discrete AND continuous. ANCOVAs are ANOVAs with a continuous x.</i>	
	P: Two-way ANOVA	aov(y ~ group * sex)	$lm(y \sim 1 + G_2 + G_3 + \dots + G_N + S_2 + S_3 + \dots + S_K + G_2 * S_2 + G_3 * S_3 + \dots + G_N * S_K)$	✓	Interaction term: changing sex changes the y ~ group parameters. <i>Note: $G_{2 \text{ to } N}$ is an indicator (0 or 1) for each non-intercept levels of the group variable. Similarly for $S_{2 \text{ to } K}$ for sex. The first line (with G_i) is main effect of group, the second (with S_j) for sex and the third is the group x sex interaction. For two levels (e.g. male/female), line 2 would just be "S_2" and line 3 would be S_2 multiplied with each G_i.</i>	[Coming]
	Counts ~ discrete x N: Chi-square test	chisq.test(groupXsex_table)	Equivalent log-linear model $glm(y \sim 1 + G_2 + G_3 + \dots + G_N + S_2 + S_3 + \dots + S_K + G_2 * S_2 + G_3 * S_3 + \dots + G_N * S_K, \text{family}=\dots)^A$	✓	Interaction term: (Same as Two-way ANOVA.) <i>Note: Run glm using the following arguments: <code>glm(model, family=poisson())</code>. As linear-model, the Chi-square test is $\log(y_i) = \log(N) + \log(\alpha_i) + \log(\beta_i) + \log(\alpha_i \beta_i)$ where α_i and β_i are proportions. See more info in the accompanying notebook.</i>	Same as Two-way ANOVA
	N: Goodness of fit	chisq.test(y)	$glm(y \sim 1 + G_2 + G_3 + \dots + G_N, \text{family}=\dots)^A$	✓	(Same as One-way ANOVA and see Chi-Square note.)	1W-ANOVA

List of common parametric (P) non-parametric (N) tests and equivalent linear models. The notation $y \sim 1 + x$ is R shorthand for $y = 1 \cdot b + a \cdot x$ which most of us learned in school. Models in similar colors are highly similar, but really, notice how similar they *all* are across colors! For non-parametric models, the linear models are reasonable approximations for non-small sample sizes (see "Exact" column and click links to see simulations). Other less accurate approximations exist, e.g., Wilcoxon for the sign test and Goodness-of-fit for the binomial test. The signed rank function is `signed_rank = function(x) sign(x) * rank(abs(x))`. The variables G_i and S_i are "[dummy coded](#)" [indicator variables](#) (either 0 or 1) exploiting the fact that when $\Delta x = 1$ between categories the difference equals the slope. Subscripts (e.g., G_2 or y_1) indicate different columns in data. `lm` requires long-format data for all non-continuous models. All of this is exposed in greater detail and worked examples at <https://lindeloev.github.io/tests-as-linear>.

^A See the note to the two-way ANOVA for explanation of the notation.

^B Same model, but with one variance per group: `gls(value ~ 1 + G2, weights = varIdent(form = ~1|group), method="ML")`.

